ALL FOUR GAUGE TRANSFORMATIONS OF THE FERMION LAGRANGIAN

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Four global gauge transformation of the free fermion Lagrangian is considered. One of these transformations has S(1) symmetry, other transformation has SU(2) symmetry, and two others have SU(3) symmetry. It is supposed, that these transformations determine four grades of the physics interactions - electromagnetic, weak, gravitational and chromatic.

◊1.INTRODUCTION

The electromagnetic, weak and strong interactions are conditioned by the local gauge symmetry of the Lagrangian. The local gauge symmetry can be gotten from the global gauge symmetry by the substitution of the constant transformation parameters by the variables. Here I will consider four global Lagrangian gauge transformations.

Use the natural metric: $\hbar = c = 1$.

◊2.

Let us consider the free fermion Lagrangian:

$$L = 0.5 \cdot i \cdot \left(\left(\partial_\mu \overline{\Psi} \right) \cdot \gamma^\mu \cdot \Psi - \overline{\Psi} \cdot \gamma^\mu \cdot (\partial_\mu \Psi) \right) - m \cdot \overline{\Psi} \cdot \Psi, \tag{1}$$

here: $0 \leq \mu \leq 3$, γ^μ are the Clifford matrices:

$$\gamma^0 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, \gamma^1 = \begin{bmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix},$$

$$\gamma^2 = \begin{bmatrix} 0 & 0 & 0 & i \\ 0 & 0 & -i & 0 \\ 0 & -i & 0 & 0 \\ i & 0 & 0 & 0 \end{bmatrix}, \gamma^3 = \begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix},$$

Ψ is the spinor, $\overline{\Psi} = \Psi^\dagger \cdot \gamma^0$ ($\overline{\Psi}$ is the adjunct for Ψ spinor).

If $\beta^\mu = \gamma^0 \cdot \gamma^\mu$ then

$$L = 0.5 \cdot i \cdot \left(\left(\partial_\mu \Psi^\dagger \right) \cdot \beta^\mu \cdot \Psi - \Psi^\dagger \cdot \beta^\mu \cdot (\partial_\mu \Psi) \right) - m \cdot \Psi^\dagger \cdot \gamma^0 \cdot \Psi. \tag{2}$$

Here:

$$\beta^1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \end{bmatrix}, \beta^2 = \begin{bmatrix} 0 & -i & 0 & 0 \\ i & 0 & 0 & 0 \\ 0 & 0 & 0 & i \\ 0 & 0 & -i & 0 \end{bmatrix}, \beta^3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Let the spinor Ψ be expressed in the following form:

$$\Psi = |\Psi| \cdot \begin{bmatrix} \exp(i \cdot g) \cdot \cos(b) \cdot \cos(a) \\ \exp(i \cdot d) \cdot \sin(b) \cdot \cos(a) \\ \exp(i \cdot f) \cdot \cos(v) \cdot \sin(a) \\ \exp(i \cdot q) \cdot \sin(v) \cdot \sin(a) \end{bmatrix}.$$

In this case the probability current vector $\overrightarrow{j}$ has got the following components:

$$j_1 = \Psi^\dagger \cdot \beta^1 \cdot \Psi =$$
$$= |\Psi|^2 \cdot \left[\cos^2(a) \cdot \sin(2 \cdot b) \cdot \cos(d-g) - \sin^2(a) \cdot \sin(2 \cdot v) \cdot \cos(q-f)\right], \tag{3}$$

$$j_2 = \Psi^\dagger \cdot \beta^2 \cdot \Psi =$$
$$= |\Psi|^2 \cdot \left[\cos^2(a) \cdot \sin(2 \cdot b) \cdot \sin(d-g) - \sin^2(a) \cdot \sin(2 \cdot v) \cdot \sin(q-f)\right], \tag{4}$$

$$j_3 = \Psi^\dagger \cdot \beta^3 \cdot \Psi = |\Psi|^2 \cdot \left[\cos^2(a) \cdot \cos(2 \cdot b) - \sin^2(a) \cdot \cos(2 \cdot v)\right]. \tag{5}$$

If

$$\rho = \Psi^\dagger \cdot \Psi, \tag{6}$$

then ρ is the probability density, i.e. $\int\int\int_{(V)} \rho(t) \cdot dV$ is the probability to find the particle with the state function Ψ in the domain V of the 3-dimensional space at the time moment t. In this case, $\{\rho, \overrightarrow{j}\}$ is the probability density $3+1$-vector.

If

$$\overrightarrow{j} = \rho \cdot \overrightarrow{u}, \tag{7}$$

then $\overrightarrow{u}$ is the average velocity for this particle.

For the left particle (for example, the left neutrino):

$a = \frac{\pi}{2}$,

$$\Psi_L = |\Psi_L| \cdot \left[\begin{array}{c} 0 \\ 0 \\ \exp(i \cdot f) \cdot \cos(v) \\ \exp(i \cdot q) \cdot \sin(v) \end{array} \right]$$

and from (3) , (4), (5), (6) and (7): $u_1^2+u_2^2+u_3^2 = 1$.Hence, the left particle velocity equals 1; hence, the mass of the left particle equals to zero.

The Clifford pentad, which contains the matrices γ^0, β^1, β^2, β^3, contains the matrix

$$\beta^4 = \begin{bmatrix} 0 & 0 & i & 0 \\ 0 & 0 & 0 & i \\ -i & 0 & 0 & 0 \\ 0 & -i & 0 & 0 \end{bmatrix},$$

else. Let us denote:

$$J_0 = \Psi^\dagger \cdot \gamma^0 \cdot \Psi, J_4 = \Psi^\dagger \cdot \beta^4 \cdot \Psi, J_0 = \rho \cdot V_0, J_4 = \rho \cdot V_4. \tag{8}$$

In this case:

$$V_0 = \sin(2 \cdot a) \cdot [\cos(b) \cdot \cos(v) \cdot \cos(g - f) + \sin(b) \cdot \sin(v) \cdot \cos(d - q)],$$

$$V_4 = \sin(2 \cdot a) \cdot [\cos(b) \cdot \cos(v) \cdot \sin(g - f) + \sin(b) \cdot \sin(v) \cdot \sin(d - q)];$$

and for every particle: $u_1^2 + u_2^2 + u_3^2 + V_0^2 + V_4^2 = 1$. Hence, for the left particles: $V_0^2 + V_4^2 = 0$.

$\diamondsuit$3.

Lagrangian is invariant for the global gauge transformation:

$$\Psi \rightarrow \exp(i \cdot \alpha) \cdot \Psi, \tag{9}$$

here α is the parameter of this gauge transformation. u_1, u_2, u_3, V_0 and V_4 are invariant for this transformation.

Let U be the weak global isospin (SU(2)) transformation with the eigenvalues $\exp(\pm i \cdot \lambda)$.

In this case for this transformation eigenvector Ψ:

$$U\Psi = |\Psi| \cdot \left[\begin{array}{c} \exp(i \cdot g) \cdot \cos(b) \cdot \cos(a) \\ \exp(i \cdot d) \cdot \sin(b) \cdot \cos(a) \\ \exp(i \cdot \lambda) \cdot \exp(i \cdot f) \cdot \cos(v) \cdot \sin(a) \\ \exp(i \cdot \lambda) \cdot \exp(i \cdot q) \cdot \sin(v) \cdot \sin(a) \end{array} \right] \tag{10}$$

(here "$\pm$" is not essential) and for $1 \leq \mu \leq 3$:

$$(U\Psi)^\dagger \cdot \beta^\mu \cdot (U\Psi) = \Psi^\dagger \cdot \beta^\mu \cdot \Psi,$$

but for $\mu = 0$ and $\mu = 4$:

$$\begin{gathered} \Psi^\dagger \cdot \gamma^0 \cdot \Psi = |\Psi|^2 \cdot \sin(2 \cdot a) \cdot \\ \left[\cos(b) \cdot \cos(v) \cdot \cos(g - f - \lambda) + \sin(b) \cdot \sin(v) \cdot \cos(d - q - \lambda)\right], \end{gathered} \tag{11}$$

$$\begin{gathered} \Psi^\dagger \cdot \beta^4 \cdot \Psi = |\Psi|^2 \cdot \sin(2 \cdot a) \cdot \\ \left[\cos(b) \cdot \cos(v) \cdot \sin(g - f - \lambda) + \sin(b) \cdot \sin(v) \cdot \sin(d - q - \lambda)\right]; \end{gathered} \tag{12}$$

Hence, the Lagrangian L is invariant for this transformation in the all its member, except the hindmost (2):

$$m \cdot \Psi^\dagger \cdot \gamma^0 \cdot \Psi, \tag{13}$$

but from (11) and (12):

$$m \cdot \left(\left(\Psi^\dagger \cdot \gamma^0 \cdot \Psi\right)^2 + \left(\Psi^\dagger \cdot \beta^4 \cdot \Psi\right)^2\right)^{0.5} \tag{14}$$

is invariant for this transformation. If the member (13) will be substituted by the expression (14) in L then L will become invariant for the weak global isospin transformation.

Let us denote:

$$O = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}.$$

In this case let the 4×4 matrices of kind:

$$\begin{bmatrix} P & O \\ O & S \end{bmatrix}$$

be denoted as the diagonal matrices, and

$$\begin{bmatrix} O & P \\ S & O \end{bmatrix}$$

be denoted as the antidiogonal matrices.

Three diagonal members of the Clifford pentad (γ^0, β^1,β^2 ,β^3 ,β^4) define the 3-dimensional space in which u_1, u_2 , u_3 are located. The physics objects move in this space. Two antidiogonal members of this pentad define the 2-dimensional space in which V_0 and V_4 are located. The weak isospin transformation acts in this space.

$\diamondsuit$4.

Let ϕ_1 ,ϕ_2 ,ϕ_3 be any real numbers (the transformation parameters); E be the identity 4×4 matrix.

Let

$$U_1 = -i \cdot \beta^3 \cdot \beta^2, Q_1 (\phi_1) = \cos (\phi_1) \cdot E + i \cdot \sin (\phi_1) \cdot U_1,$$

$$U_2 = -i \cdot \beta^3 \cdot \beta^1, Q_2 (\phi_2) = \cos (\phi_2) \cdot E + i \cdot \sin (\phi_2) \cdot U_2,$$

$$U_3 = -i \cdot \beta^1 \cdot \beta^2, Q_3 (\phi_3) = \cos (\phi_3) \cdot E + i \cdot \sin (\phi_3) \cdot U_3.$$

In this case the Lagrangian L is invariant for the following transformations:

$$\left\{\begin{array}{c}\beta^1 \to \beta^1,\\ \beta^2 \to \beta^2 \cdot \cos\left(2 \cdot \phi_1\right) + \beta^3 \cdot \sin\left(2 \cdot \phi_1\right),\\ \beta^3 \to \beta^3 \cdot \cos\left(2 \cdot \phi_1\right) - \beta^2 \cdot \sin\left(2 \cdot \phi_1\right),\\ \Psi \to Q_1\left(\phi_1\right) \cdot \Psi\\ \gamma^0 \to \gamma^0,\\ \beta^4 \to \beta^4.\end{array}\right\} \tag{15}$$

$$\left\{\begin{array}{c}\beta^1 \to \beta^1 \cdot \cos\left(2 \cdot \phi_2\right) + \beta^3 \cdot \sin\left(2 \cdot \phi_2\right),\\ \beta^2 \to \beta^2,\\ \beta^3 \to \beta^3 \cdot \cos\left(2 \cdot \phi_2\right) - \beta^1 \cdot \sin\left(2 \cdot \phi_2\right),\\ \Psi \to Q_2\left(\phi_2\right) \cdot \Psi\\ \gamma^0 \to \gamma^0,\\ \beta^4 \to \beta^4.\end{array}\right\} \tag{16}$$

$$\left\{\begin{array}{c}\beta^1 \to \beta^1 \cdot \cos\left(2 \cdot \phi_3\right) - \beta^2 \cdot \sin\left(2 \cdot \phi_3\right),\\ \beta^2 \to \beta^2 \cdot \cos\left(2 \cdot \phi_3\right) + \beta^1 \cdot \sin\left(2 \cdot \phi_3\right),\\ \beta^3 \to \beta^3,\\ \Psi \to Q_3\left(\phi_3\right) \cdot \Psi\\ \gamma^0 \to \gamma^0,\\ \beta^4 \to \beta^4.\end{array}\right\} \tag{17}$$

Hence, these transformations coordinate to the turning in the 3-dimensional space of β^1, β^2, β^3.

$\diamondsuit$5.

By analogy with (15), (16), (17): let

$$U_0 = -i \cdot \gamma^0 \cdot \beta^4, Q_0\left(\phi\right) = \cos\left(\phi\right) \cdot E + i \cdot \sin\left(\phi\right) \cdot U_0. \tag{18}$$

In this case, Q_0 coordinates to the turning in the 2-dimensional space V_0, V_4 (8). Because

$$Q_0\left(\phi\right) = \begin{bmatrix} \exp\left(-i \cdot \phi\right) & 0 & 0 & 0\\ 0 & \exp\left(-i \cdot \phi\right) & 0 & 0\\ 0 & 0 & \exp\left(i \cdot \phi\right) & 0\\ 0 & 0 & 0 & \exp\left(i \cdot \phi\right)\end{bmatrix},$$

then the product of this transformation and the transformation (9) with the parameter ϕ: $\exp\left(i \cdot \phi\right) \cdot Q_0\left(\phi\right)$ is the weak global isospin transformation

(10) with the parameter $2 \cdot \phi$. Here $Q_0(\phi)$ is the symmetric transformation but $\exp(i \cdot \phi)$ disturbs this symmetry.

♢6.

Let

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

and

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

be the Pauli matrices.

Six Clifford's pentads exists, only:

the red pentad ζ:

$$\zeta^1 = \begin{bmatrix} \sigma_1 & O \\ O & -\sigma_1 \end{bmatrix}, \zeta^2 = \begin{bmatrix} \sigma_2 & O \\ O & \sigma_2 \end{bmatrix}, \zeta^3 = \begin{bmatrix} -\sigma_3 & O \\ O & -\sigma_3 \end{bmatrix},$$

$$\gamma_\zeta^0 = \begin{bmatrix} O & -\sigma_1 \\ -\sigma_1 & O \end{bmatrix}, \zeta^4 = -i \cdot \begin{bmatrix} O & \sigma_1 \\ -\sigma_1 & O \end{bmatrix};$$

the green pentad η:

$$\eta^1 = \begin{bmatrix} -\sigma_1 & O \\ O & -\sigma_1 \end{bmatrix}, \eta^2 = \begin{bmatrix} \sigma_2 & O \\ O & -\sigma_2 \end{bmatrix}, \eta^3 = \begin{bmatrix} -\sigma_3 & O \\ O & -\sigma_3 \end{bmatrix},$$

$$\gamma_\eta^0 = \begin{bmatrix} O & -\sigma_2 \\ -\sigma_2 & O \end{bmatrix}, \eta^4 = i \cdot \begin{bmatrix} O & \sigma_2 \\ -\sigma_2 & O \end{bmatrix};$$

the blue pentad θ:

$$\theta^1 = \begin{bmatrix} -\sigma_1 & O \\ O & -\sigma_1 \end{bmatrix}, \theta^2 = \begin{bmatrix} \sigma_2 & O \\ O & \sigma_2 \end{bmatrix}, \theta^3 = \begin{bmatrix} \sigma_3 & O \\ O & -\sigma_3 \end{bmatrix},$$

$$\gamma_\theta^0 = \begin{bmatrix} O & -\sigma_3 \\ -\sigma_3 & O \end{bmatrix}, \theta^4 = -i \cdot \begin{bmatrix} O & \sigma_3 \\ -\sigma_3 & O \end{bmatrix};$$

the light pentad β:

$$\beta^1 = \begin{bmatrix} \sigma_1 & O \\ O & -\sigma_1 \end{bmatrix}, \beta^2 = \begin{bmatrix} \sigma_2 & O \\ O & -\sigma_2 \end{bmatrix}, \beta^3 = \begin{bmatrix} \sigma_3 & O \\ O & -\sigma_3 \end{bmatrix},$$

$$\gamma^0 = \begin{bmatrix} O & I \\ I & O \end{bmatrix}, \beta^4 = i \cdot \begin{bmatrix} O & I \\ I & O \end{bmatrix};$$

the sweet pentad $\underline{\Delta}$:

$$\underline{\Delta}^1 = \begin{bmatrix} O & -\sigma_1 \\ -\sigma_1 & O \end{bmatrix}, \underline{\Delta}^2 = \begin{bmatrix} O & -\sigma_2 \\ -\sigma_2 & O \end{bmatrix}, \underline{\Delta}^3 = \begin{bmatrix} O & -\sigma_3 \\ -\sigma_3 & O \end{bmatrix},$$

$$\underline{\Delta}^0 = \begin{bmatrix} -I & O \\ O & I \end{bmatrix}, \underline{\Delta}^4 = i \cdot \begin{bmatrix} O & I \\ -I & O \end{bmatrix};$$

the bitter pentad $\underline{\Gamma}$:

$$\underline{\Gamma}^1 = i \cdot \begin{bmatrix} O & -\sigma_1 \\ \sigma_1 & O \end{bmatrix}, \underline{\Gamma}^2 = i \cdot \begin{bmatrix} O & -\sigma_2 \\ \sigma_2 & O \end{bmatrix}, \underline{\Gamma}^3 = i \cdot \begin{bmatrix} O & -\sigma_3 \\ \sigma_3 & O \end{bmatrix},$$

$$\underline{\Gamma}^0 = \begin{bmatrix} -I & O \\ O & I \end{bmatrix}, \underline{\Gamma}^4 = \begin{bmatrix} O & I \\ I & O \end{bmatrix}.$$

The average velocity vector for the sweet pentad has gotten the following components:

$$V_0^{\underline{\Delta}} = -\cos(2 \cdot a),$$

$$V_1^{\underline{\Delta}} = -\sin(2 \cdot a) \cdot [\cos(b) \cdot \sin(v) \cos(g - q) + \sin(b) \cdot \cos(v) \cos(d - f)],$$

$$V_2^{\underline{\Delta}} = -\sin(2 \cdot a) \cdot [-\cos(b) \cdot \sin(v) \sin(g - q) + \sin(b) \cdot \cos(v) \sin(d - f)],$$

$$V_3^{\underline{\Delta}} = -\sin(2 \cdot a) \cdot [\cos(b) \cdot \cos(v) \cos(g - f) - \sin(b) \cdot \sin(v) \cos(d - q)],$$

$V_4^{\underline{\Delta}} = -\sin(2 \cdot a) \cdot [-\cos(b) \cdot \cos(v) \sin(g - f) - \sin(b) \cdot \sin(v) \sin(d - q)]$.

Therefore, here the antidiogonal matrices $\underline{\Delta}^1$ and $\underline{\Delta}^2$ define the 2-dimensional space ($V_1^{\underline{\Delta}}$, $V_2^{\underline{\Delta}}$) in which the weak isospin transformation acts. The antidiogonal matrices $\underline{\Delta}^3$ and $\underline{\Delta}^4$ define similar space ($V_3^{\underline{\Delta}}$, $V_4^{\underline{\Delta}}$). The sweet pentad is kept a single diagonal matrix, which defines the one-dimensional space ($V_0^{\underline{\Delta}}$) for the moving of the objects.

Like the sweet pentad, the bitter pentad with four antidiogonal matrices and with single diagonal matrix defines two 2-dimensional spaces, in which the weak isospin transformation acts, and single one-dimensional space for the moving of the objects.

$\diamondsuit$7.

Each colored pentad with 3 diagonal matrices and with 2 antidiogonal matrices, like the light pentad, defines single 2-dimensional space, in which the weak isospin interaction acts, and defines single 3-dimensional space for the moving the physics objects.

So in the Lagrangian (2) the matrices of the light pentad are to be replaced by the elements from some colored pentad. Hence probably, the free fermion Lagrangian has of the form of:

$$\begin{aligned} L = 0.5 \cdot i \cdot \left(\left(\partial_\mu \Psi^\dagger \right) \cdot \kappa^\mu \cdot \Psi - \Psi^\dagger \cdot \kappa^\mu \cdot \left(\partial_\mu \Psi \right) \right) - \\ m \cdot \left(\left(\Psi^\dagger \cdot \gamma_\kappa^0 \cdot \Psi \right)^2 + \left(\Psi^\dagger \cdot \kappa^4 \cdot \Psi \right)^2 \right)^{0.5} \end{aligned} \tag{19}$$

here γ_κ^0, κ^1, κ^2, κ^3, κ^4 are the members or of the light pentad or of a colored pentad.

By analogy with (15),(16),(17): let

$$\Lambda_\theta = i \cdot \zeta^0 \cdot \eta^0, G_\theta(\phi_\theta) = \cos(\phi_\theta) \cdot E + i \cdot \sin(\phi_\theta) \cdot \Lambda_\theta. \tag{20}$$

In this case the Lagrangian (19) is invariant for the following transformation:

$\Psi \to G_\theta(\phi_\theta) \cdot \Psi$,
$\zeta^0 \to \zeta^0 \cdot \cos(2 \cdot \phi_\theta) + \eta^0 \cdot \sin(2 \cdot \phi_\theta)$,
$\eta^0 \to \eta^0 \cdot \cos(2 \cdot \phi_\theta) - \zeta^0 \cdot \sin(2 \cdot \phi_\theta)$,
$\zeta^4 \to \zeta^4 \cdot \cos(2 \cdot \phi_\theta) - \eta^4 \cdot \sin(2 \cdot \phi_\theta)$,

$\eta^4 \to \eta^4 \cdot \cos(2 \cdot \phi_\theta) + \zeta^4 \cdot \sin(2 \cdot \phi_\theta),$
$\zeta^1 \to \zeta^1 \cdot \cos(2 \cdot \phi_\theta) + \eta^2 \cdot \sin(2 \cdot \phi_\theta),$
$\eta^2 \to \eta^2 \cdot \cos(2 \cdot \phi_\theta) - \zeta^1 \cdot \sin(2 \cdot \phi_\theta),$
$\zeta^2 \to \zeta^2 \cdot \cos(2 \cdot \phi_\theta) + \eta^1 \cdot \sin(2 \cdot \phi_\theta),$
$\eta^1 \to \eta^1 \cdot \cos(2 \cdot \phi_\theta) - \zeta^2 \cdot \sin(2 \cdot \phi_\theta),$
$\eta^3 \to \zeta^3,$
$\zeta^3 \to \eta^3,$
$\theta^0 \to \theta^0,$
$\theta^4 \to \theta^4,$
$\theta^3 \to \theta^3,$
$\theta^1 \to \theta^1 \cdot \cos(2 \cdot \phi_\theta) - \theta^2 \cdot \sin(2 \cdot \phi_\theta),$
$\theta^2 \to \theta^2 \cdot \cos(2 \cdot \phi_\theta) + \theta^1 \cdot \sin(2 \cdot \phi_\theta),$
$\beta^0 \to \beta^0,$
$\beta^4 \to \beta^4,$
$\beta^3 \to \beta^3,$
$\beta^1 \to \beta^1 \cdot \cos(2 \cdot \phi_\theta) + \beta^2 \cdot \sin(2 \cdot \phi_\theta),$
$\beta^2 \to \beta^2 \cdot \cos(2 \cdot \phi_\theta) - \beta^1 \cdot \sin(2 \cdot \phi_\theta).$

Therefore, this transformation corresponds to a turning in the space of the red and the green pentads. Similarly this, the transformations with

$$\Lambda_\zeta = i \cdot \theta^0 \cdot \eta^0, G_\zeta(\phi_\zeta) = \cos(\phi_\zeta) \cdot E + i \cdot \sin(\phi_\zeta) \cdot \Lambda_\zeta. \tag{21}$$

corresponds to a turning in the space of the green and the blue pentads, and the transformation with

$$\Lambda_\eta = i \cdot \theta^0 \cdot \zeta^0, G_\eta(\phi_\eta) = \cos(\phi_\eta) \cdot E + i \cdot \sin(\phi\eta) \cdot \Lambda_\eta. \tag{22}$$

corresponds to a turning in the space of red and blue pentads.

But similar transformation, which corresponds to a turning in the space of a colored and the light pentads, does not exist.

RESUME

Therefore, the transformation (9) is the transformation with the invariance U(1). The transformation (10) is the transformation with the invariance SU(2). The transformations (15), (16), (17) are the transformations with the invariance SU(3) in the space of the diagonal matrices of the same pentad. The transformations with (20), (21), (22) are the transformation with invariance SU(3), too, but in the space of the colored pentads. I'm assume that these transformations (with (20), (21), (22)) determine the quarks

interaction. And because the transformations (9), (10) coordinate to the electroweak interaction then the transformations (15), (16), (17) will remain for the gravitation.

That is to say we have got four gauge transformations for four kinds of the physics interactions: electromagnetic, weak, gravitation, strong.

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